UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

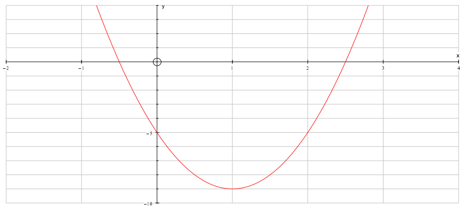
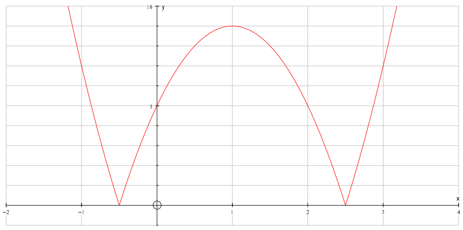
PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 $x^2+(2k+10)x+(k^2+5)=0$ $(2k+10)^2=4(k^2+5)$ $k=-2$	M1 M1 A1 [3]	M1 for equating to zero and use of $b^2=4ac$ M1 for solution
(or $\frac{dy}{dx}=2x+(2k+10), x=-(k+5)$ $0=(k+5)^2-(2k+10)(k+5)+k^2+5$ leading to $k=-2$)	M1 M1 A1	M1 for differentiation and attempt to equate to zero. M1 for attempt to substitute in for x in terms of k, for $y=0$ and for attempt at solution.
(or $(x+A)^2=x^2+(2k+10)x+k^2+5$ $A=(k+5), A^2=k^2+5$ $(k+5)^2=k^2+5$, leading to $k=-2$)	M1 M1 A1	M1 for approach M1 for equating and attempt at solution
(or by completing the square $y=(x+(k+5))^2-(k+5)^2+(k^2+5)$ $(k+5)^2=k^2+5$ leading to $k=-2$)	M1 M1 A1	M1 for approach M1 for equating last 2 terms to zero and attempt to solve
2 ${}^5C_3 2^2 a^3=(10)\,{}^4C_2\frac{a^2}{9}$ $a=\frac{1}{6}$	B1B1 M1 A1 [4]	B1 for ${}^5C_3 2^2 a^3$, B1 for ${}^4C_2\frac{a^2}{9}$ M1 for a relationship between the 2 coefficients and attempt to solve
3 **(a)** $k=2, m=3, p=1$ **(b)** **(i)** 5 **(ii)** $\frac{2\pi}{3}$	B3 B1 B1 [5]	B1 for each
There must be evidence of working without a calculator in all parts **4** **(i)** $\frac{(4+\sqrt{2})}{(1+\sqrt{2})}\frac{(1-\sqrt{2})}{(1-\sqrt{2})}=2\sqrt{2}$ **(ii)** Area $=\frac{1}{2}\times(4+2\sqrt{2})\times(1+\sqrt{2})$ $=4+3\sqrt{2}$ **(iii)** Area = AC^2 $=(4+2\sqrt{2})^2+(1+\sqrt{2})^2$ $=27+18\sqrt{2}$	M1A1 M1 A1 M1 A1 [6]	M1 for attempt to rationalise and attempt to expand M1 for attempt at area using surd form and attempt to expand M1 for attempt at AC^2 or AC in surd form, with attempt to expand

5 **(i)** $2\left(\frac{1}{8}\right)-5\left(\frac{1}{4}\right)+10\left(\frac{1}{2}\right)-4$	M1	M1 for substitution of $x = 0.5$ or attempt at long division
$= 0$	A1	
(ii) $(2x-1)(x^2-2x+4)$	M1A1	M1 attempt to obtain quadratic factor A1 for correct quadratic factor
For (x^2-2x+4), '$b^2 < 4ac$'	M1	M1 for correct use of discriminant or solution of quadratic equation = 0
so only one real root of $x = 0.5$	A1 [6]	A1, all correct with statement of root.
6 **(i)** $\lg y - 3 = \frac{1}{5}(x-5)$	B1M1 A1	B1 for gradient, M1 for use of straight line equation
(ii) Either $b = \frac{1}{5}$	B1	B1 for $b = \frac{1}{5}$
$y = 10^{\left(\frac{1}{5}x+2\right)}$, $= 10^{\frac{1}{5}x}10^2$	M1	M1 for use of powers of 10 correctly to obtain a
$a = 100$	A1 [6]	A1 for a
Or $\lg y = \lg a + \lg 10^{bx}$ $\lg y = \lg a + bx$, $\lg a = 2$	M1	M1 for use of logarithms correctly to obtain a
$a = 100$	A1	A1 for a
$b = \frac{1}{5}$	B1	B1 for $b = \frac{1}{5}$
Or $10^3 = a(10)^{5b}$ $10^5 = a(10)^{15b}$	M1	M1 for simultaneous equations involving powers of 10
$b = \frac{1}{5}$, $a = 100$	B1, A1	B1 for $b = \frac{1}{5}$, A1 for $a = 100$
7 **(i)** ${}^{14}C_6 = 3003$	B1	
(ii) ${}^{8}C_4 \times {}^{6}C_2$ $= 1050$	B1B1 B1	B1 for ${}^{8}C_4$ or ${}^{6}C_2$ B1 for × by ${}^{6}C_2$ or ${}^{8}C_4$ B1 for 1050
(iii) ${}^{8}C_6 + 6\,{}^{8}C_5 = 364$	B1B1 B1 [7]	B1 for ${}^{8}C_6$ or equivalent B1 for $6\,{}^{8}C_5$ or equivalent B1 for 364

8 **(i)** 	B1 B1 B1 B1	B1 for $x = -0.5$ B1 for $x = 2.5$ B1 for $y = -5$ B1 for shape
(ii) $(1, -9)$	B1	
(iii) 	√B1 B1 [7]	√B1 on shape from (i) B1 for a completely correct sketch
9 **(i)** $\Delta OBA : \theta + 2\left(\frac{\theta}{3}\right) = \pi$	M1 A1	M1 for using angles in an isosceles triangle
(ii) $9\pi = r \times \frac{3\pi}{5}$ $r = 15$	M1 A1	M1 for use of $s = r\theta$
(iii) Area $= \left(\frac{1}{2} \times 15^2 \times \frac{3\pi}{5}\right) - \left(\frac{1}{2} \times 15^2 \times \sin\frac{3\pi}{5}\right)$ $=105$	M1M1 A1 [7]	M1 for use of $\frac{1}{2}r^2\theta$ or $\frac{1}{2}rs$ M1 for use of $\frac{1}{2}r^2 \sin\theta$ or other correct method
10 **(i)** $\begin{pmatrix} 29 \\ -13 \end{pmatrix} - \begin{pmatrix} 5 \\ -6 \end{pmatrix} = \begin{pmatrix} 24 \\ -7 \end{pmatrix}$	M1	M1 for subtraction
Magnitude = 25, unit vector $\frac{1}{25}\begin{pmatrix} 24 \\ -7 \end{pmatrix}$	M1 A1	M1 for attempt to find magnitude of their vector
(ii) $2\overrightarrow{AC} = 3\overrightarrow{AB}$ or $2\overrightarrow{AB} + 2\overrightarrow{BC} = 3\overrightarrow{AB}$ leading to $\overrightarrow{AC} = \begin{pmatrix} 36 \\ -10.5 \end{pmatrix}$	M1	M1 for attempt to find $\overrightarrow{AC}$ – may be part of a larger method
$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ or $\overrightarrow{OB} - \overrightarrow{OA} = 2\overrightarrow{OC} - 2\overrightarrow{OB}$	M1	M1 for attempt to find $\overrightarrow{OC}$
leading to $\overrightarrow{OC} = \begin{pmatrix} 41 \\ -16.5 \end{pmatrix}$ (equivalent methods acceptable)	A1 A1 [7]	A1 for each

11 **(i)** $2\operatorname{cosec}^2 x - 5\operatorname{cosec} x - 3 = 0$	M1A1	M1 for use of correct identity or to get in terms of sin x
$(2\operatorname{cosec}\theta + 1)(\operatorname{cosec}\theta - 3) = 0$	DM1	DM1 for attempt to solve
leading to $\sin x = \frac{1}{3}$, $x = 19.5°, 160.5°$	A1√A1	√ $180^\circ -$ their x
(ii) $\tan 2y = \frac{5}{4}$	M1	M1 for attempt to get in terms of tan
$2y = 51.34°, 231.34°$	M1	M1 for dealing correctly with double angle
$y = 25.7°, 115.7°$	A1,√A1	√ 90° their y
(iii) $\left(z + \frac{\pi}{6}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}$ $z = \frac{2\pi}{3} - \frac{\pi}{6}$ $\left(\frac{4\pi}{3} - \frac{\pi}{6}\right)$	M1	M1 for dealing with order correctly and attempt to solve
$z = \frac{\pi}{2}, \frac{7\pi}{6}$ allow 1.57, 3.67	A1, A1 [12]	
12 **EITHER**		
(i) $\frac{dy}{dx} = 9x^2 + 4x - 5$ when x = –1, $\frac{dy}{dx} = 0$	M1	M1 for differentiation and substitution of $x = -1$
tangent $y = 5$, A (0, 5)	DM1 A1	DM1 for attempt at equation of tangent and coordinates of A
(ii) B (0, 1) At B, $\frac{dy}{dx} = -5$	B1	B1 for B
normal $y - 1 = \frac{1}{5}x$ C (–5, 0)	M1A1	M1 for attempt at normal and C, must be from differentiation and using correct point
At D $\frac{1}{5}x + 1 = 5$, D (20, 5)	M1A1	M1 for attempt to obtain D, equating normal and tangent equations
Area = $\frac{1}{2} \times 20 \times 5$,	M1	M1 for valid attempt at area
= 50	A1 [10]	

12 OR		
$\dfrac{dy}{dx} = 3x^2 - 12x + 9$	M1	M1 for differentiation and equating to 0, can be using a product
When $\dfrac{dy}{dx} = 0$, $x = 1, 3$	M1	M1 for attempt to solve
	A1	A1 for both x values
$P(1, 4)$	A1	A1 for y coordinate
Area = $8 - \int_1^3 x^3 - 6x^2 + 9x \, dx$	√B1M1	√B1 on y coordinate for area of rectangle M1 for attempt to integrate
$= 8 - \left[\dfrac{x^4}{4} - 2x^3 + \dfrac{9x^2}{2}\right]_1^3$	A2,1,0	–1 each error
$= 8 - \dfrac{27}{4} + \dfrac{11}{4}$	DM1	DM1 for application of limits
$= 4$	A1 [10]	